1. Let . Suppose that is a function.

⊆ Let . That means . By the definition of the inverse relation, . So .

⊇ Let . That means . By the definition of the inverse relation, . So .

Therefore .

⊆ Let . That means . By the definition of the inverse relation, . So .

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Therefore .

1. Let . That is,

Assume that is one-to-one and that and . By the definition of the inverse relation, that means that and . That means and . Since equals is transitive, . Since is one-to-one, .

Therefore is a function.

Let . That is, there is an element such that and . That means and . By definition of the inverse relation, and . Since is a function, then .

Therefore, is one-to-one.

⊆ Let . That means . By the definition of the inverse relation, . So .

⊇ Let . That means . By the definition of the inverse relation, . So .

So, .

Therefore is onto.

1. Let be the set of people at the party. Suppose that . Let , where and the number of people that person p knows. By the definition of the problem, there is no case where one person, , knows another person, , but does not know . So if there is someone that knows nobody, then there is nobody that knows everybody. So, and cannot exist together in , thus . Since then is not one-to-one, by the pigeonhole principle. Now let and be two distinct people at the party (elements of) for which This means that and are two people that the party who know the exact same number of people.
2. 1. 1. There are functions from the set of students in the class to the set of scores.
      2. There are functions that are one-to-one.
      3. There are functions that are onto.
   2. 1. There are functions from the set of students in the class to the set of letter grades.
      2. There are 0 functions that are one-to-one.
      3. There are functions that are onto.
3. Let be the set of customers, let be the set of combo meals, and let . We know that . Since there are burgers, sides, and drinks, . So , which means is not one-to-one, by the pigeonhole principle. Now let and be distinct customers for which . This means that and are two different customers that has the exact same meal.